Mitigating Pilot Contamination by Pilot Reuse and Power Control Schemes for Massive MIMO Systems

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Abstract—The performance of massive multiple input multiple output systems may be limited by inter-cell pilot contamination (PC) unless appropriate PC mitigation or avoidance schemes are employed. In this paper we develop techniques based on existing long term evolution (LTE) measurements - open loop power control (OLPC) and pilot sequence reuse schemes, that avoid PC within a group of cells. We compare the performance of simple least-squares channel estimator with the higher-complexity minimum mean square error estimator, and evaluate the performance of the recently proposed coordinated pilot allocation (CPA) technique (which is appropriate in cooperative systems). The performance measures of interest include the normalized mean square error of channel estimation, the downlink signalto-interference-plus-noise and spectral efficiency when employing maximum ratio transmission or zero forcing precoding at the base station. We find that for terminals moving at vehicular speeds, PC can be effectively mitigated in an operation and maintenance node using both the OLPC and the pilot reuse schemes. Additionally, greedy CPA provides performance gains only for a fraction of terminals, at the cost of degradation for the rest of the terminals and higher complexity. These results indicate that in practice, PC may be effectively mitigated without the need for second-order channel statistics or inter-cell cooperation.

I. INTRODUCTION

A large excess of base station (BS) antennas over the number or served terminals has been shown to provide attractive spectral efficiency gains [1] with time-division duplex (TDD) operation and channel knowledge at the BS. The channel coherence is typically constrained in time as well as frequency, leading to a trade-off between the resources spent on uplink pilot symbols for channel estimation and those available for data symbols. The pilot overhead can be reduced by reusing pilot sequences in nearby cells. However, pilot reuse potentially causes corruption of the channel estimates, referred to as pilot contamination (PC) or pilot pollution. PC has been shown to limit the achievable performance of non-cooperative multiuser multiple input multiple output (MU MIMO) systems [1], [2]. Specifically, it has been found that PC may cause the saturation of the signal-to-interference-plus-noise ratio (SINR) as the number of BS antennas M increases to ∞ , while the SINR increases approximately linearly with M in the absence of PC [2]. More precisely, as has been pointed out by [3], when the number of users is comparable to the number of antennas, the performance of a simple matched filter with contaminated estimate is limited by the pilot interference. These insights triggered the research community to find effective measures to mitigate the impact of PC both in the non-asymptotic and asymptotic regimes [4], [5], [6], [7], [8], [9], [10].

A pilot contamination precoding scheme is proposed by [5], according to which each BS linearly combines messages aimed to terminals of different cells that reuse the same pilot sequence. This limited collaboration between BSs can resolve the pilot contamination problem and allows for tight pilot reuse. Another approach to improve the channel estimation is exploiting second order channel statistics using a Bayesian estimator, that mitigates PC for spatially well-separated users [6]. However, exploiting second order channel statistics entails the overhead of estimating the covariance matrices and computation complexity. An iterative filter may be employed that avoids explicit estimation of covariance matrices, but its convergence is still an open problem [11]. A low-complexity Bayesian channel estimator, coined Polynomial Expansion Channel is proposed by [7], that is shown to be efficient in the presence of PC. A limited cooperation, based on the exchange of second order channel statistics and making use of Bayesian channel estimation is proposed by [6], in which it is claimed that PC can be almost completely eliminated by allocating pilots to spatially well-separated terminals. A different approach based on a less aggressive pilot reuse is proposed in [4] which can be effectively combined by the spatial separation of users in different cells [8]. The spatial and temporal characteristics of the channels can in fact be exploited to separate users that reuse the same pilot sequence in neighbor cells [9]. Finally, the series of works represented by [10] show that pilot contamination can be eliminated by blind pilot decontamination using non linear receivers and power control.

In this paper, we propose a low-complexity pilot power control (PPC) and pilot reuse schemes within the framework of current LTE measurements. We perform channel estimation with a least-squares (LS) estimator and benchmark it against the performance of Bayesian estimator that assumes perfect knowledge of all cross-channel covariance matrices at the BS. We show that these schemes effectively mitigate PC and approach perfect SINR performance in the downlink without the overhead and complexity of Bayesian estimation and, in terms of DL spectral efficiency, outperform it when zeroforcing (ZF) precoding is used at the BS. Next, we investigate coordination-based improvements to the Bayesian channel estimation proposed by [6] and compare it against the performance

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of non-cooperative schemes. Surprisingly, in contrast to results previously reported and widely believed in the literature [4], [8], [9], we find that the greedy pilot allocation algorithm provides gains only for the initial few pilot allocations, at the cost of increased PC and hence lower performance for the terminals that are allocated pilots in latter iterations.

The next section describes our system model that includes the description of the pilot based channel estimation algorithms, the DL signal model and spectral efficiency calculation. Section III describes two practical PC mitigation schemes that do not require BS cooperation and the CPA based pilot decontamination method. Section IV presents numerical results and Section V concludes the paper.

II. SYSTEM MODEL

A. Pilot Signal Model and Channel Estimation

In this work we assume reciprocity based channel estimation by means of uplink pilot sequences, as in [1], [4], [6] and [8]. The k^{th} terminal of Cell-*j* transmits a pilot sequence \mathbf{s}_k comprising of τ symbols:

$$\mathbf{s}_{k} = [s_{k,1}, s_{k,2}, \dots, s_{k,\tau}]^{T};$$
$$\sum_{i} ||\mathbf{s}_{k,i}|| = \tau P_{T},$$

where P_T is the uplink transmit power (assumed being equal for all symbols within the pilot sequence). Then the $M \times \tau$ received signal matrix at BS-*j* due to the transmission of the k^{th} pilot sequence becomes:

$$\mathbf{X}_{jk} = \sum_{l \in \mathcal{J}} \mathbf{g}_{jkl} \mathbf{s}_k^T + \mathbf{N}_j,$$

where \mathcal{J} is the set of indexes of the cells of the entire system and \mathbf{g}_{jkl} is the $M \times 1$ channel vector between the k^{th} terminal of the l^{th} cell and the j^{th} BS and \mathbf{N}_j is the $M \times \tau$ thermal noise at the j^{th} BS. Since we assume that the pilot sequences within a cell are orthogonal, we ignore the impact of intracell pilot interference due to the simultaneous reception of other pilot sequences $\mathbf{s}_i, i \neq k$. Notice that given a total budget of N_{coh} number of symbols, there is an inherent trade-off between the number of $\tau = N_{pilot}$ symbols used for channel estimation and the number of symbols available for data transmission.

BS-*j* estimates the uplink channel using either LS or minimum mean square error (MMSE) channel estimation and, assuming channel reciprocity, uses the channel estimate to form the DL precoding matrix discussed in the next subsection:

$$\hat{\mathbf{g}}_{jk}^{\mathrm{LS}} = \mathbf{g}_{jk} + \sum_{l \in J, l \neq j} \mathbf{g}_{jkl} + \frac{\mathbf{N}_j \mathbf{s}_k^{-}}{\tau P_T},$$

and

$$\hat{\mathbf{g}}_{jk}^{\text{MMSE}} = \mathbf{R}_{jk} \left(\sigma^2 \mathbf{I}_M + \tau \sum_{l \in \mathcal{J}} \mathbf{R}_{jkl} \right)^{-1} \mathbf{S}_k^H \mathbf{x}_{jk},$$
where $\mathbf{x}_{jk} = \text{vec} (\mathbf{X}_{jk}) \in \mathbb{C}^{M \tau \times 1},$
and $\mathbf{S}_k = \mathbf{s}_k \otimes \mathbf{I}_M \in \mathbb{C}^{M \tau \times M},$

where $\mathbf{R}_{jk} \triangleq \mathbf{R}_{jkk}$ is the $M \times M$ covariance matrix of the channel between the j^{th} BS and the k^{th} served terminal in that cell, while \mathbf{R}_{jkl} is between the j^{th} BS and the k^{th} terminal in that the l^{th} cell and \otimes denotes the Kronecker product.

B. Downlink Signal Model

In this paper we focus on the DL performance in terms of the mean square error (MSE) of the channel estimation, the achieved DL signal-to-interference-plus-noise-ratio (SINR) and the resulting sum rate. To this end, we consider the following DL signal model of the j^{th} cell:

$$\mathbf{y}_{j} = \sqrt{\frac{P_{BS}}{MK}} [\mathbf{w}_{j1}, \mathbf{w}_{j2}, \dots, \mathbf{w}_{jK}] [a_{j1}, a_{j2}, \dots, a_{jK}]^{T},$$
$$\mathbf{W}_{j} = [\mathbf{w}_{j1}, \mathbf{w}_{j2}, \dots, \mathbf{w}_{jK}],$$
$$\sum_{k=1}^{K} ||\mathbf{w}_{jk}|| = 1,$$

where P_{BS} denotes the transmit power of the BS, the transmitted data symbols are of unit power and ||.|| denotes the Euclidean norm of a vector and the Frobenius norm of a matrix. In this paper we consider the well known maximum ratio (MRT) or ZF transmissions on the DL, that is we use the following weight vectors:

$$\mathbf{w}_{jk}^{\mathrm{MRT}} = rac{\hat{\mathbf{g}}_{jk}^{*}}{||\hat{\mathbf{G}}_{j}||} \quad ext{and} \quad \mathbf{w}_{jk}^{\mathrm{ZF}} = rac{\hat{\mathbf{g}}_{jk}^{\dagger}}{||\hat{\mathbf{G}}_{j}^{\dagger}||},$$

where $\hat{\mathbf{g}}_{jk}^{\dagger}$ is the k^{th} column of the pseudo-inverse matrix $\hat{\mathbf{G}}_{j}^{\dagger}$, given by $\hat{\mathbf{G}}_{j}^{\dagger} = \left(\hat{\mathbf{G}}_{j}^{H}\hat{\mathbf{G}}_{j}\right)^{-1}\hat{\mathbf{G}}_{j}^{H}$.

C. Performance Measures of Interest

Our first performance measure of interest is the normalized mean square error of the channel estimate (NMSE), defined as [6]:

$$\eta_{jk} \triangleq \frac{\mathcal{M}_{jk}}{\mathbb{E}\{||\mathbf{g}_{jk}||^2\}} = \frac{\mathbb{E}||\hat{\mathbf{g}}_{jk} - \mathbf{g}_{jk}||^2}{\mathbb{E}\{||\mathbf{g}_{jk}||^2\}}.$$

The second performance measure of interest is the DL SINR and resulting spectral efficiency, which can be calculated as:

$$\Gamma_{jk} = \frac{|\mathbf{g}_{jk}^T \mathbf{w}_{jk}|^2}{\sum_{i \neq j} |\mathbf{g}_{jk}^T \mathbf{w}_{ji}|^2 + \sum_{l \neq j} \sum_{i=1}^K |\mathbf{g}_{lk}^T \mathbf{w}_{li}|^2 + \frac{\sigma_n^2 M K}{P_{BS}}}$$

and

$$\mathcal{R}_{jk} = \left(\frac{N_{coh} - N_{pilot}}{N_{coh}}\right) \left(\frac{T_s - T_{CP}}{T_s}\right) \log_2\left(1 + \Gamma_{jk}\right) \text{bps/Hz}$$

where – assuming orthogonal frequency division modulation – we explicitly take into account the overhead within the symbol time (T_s) for transmitting a cyclic prefix (T_{CP}) .

III. PILOT CONTAMINATION AVOIDING AND MITIGATION SCHEMES

A. Pilot Power Control (PPC)

Pilot power control aims at mitigating the impact of pilot contamination by reducing the transmit power of users that are relatively close to their serving BSs. This technique has proven to be efficient to reduce the level of intercell interference for data channels in multicell long term evolution (LTE) systems [12]. Therefore, when applied to control the transmit power of pilot signals, the LTE open loop path loss compensating power control (OLPC) scheme appears as a natural candidate for mitigating pilot contamination as well. A key characteristic of the LTE OLPC is that it only requires the large scale fading (path loss) between the served user and its base station as its key input parameter. When LTE OLPC is employed for PPC, it operates as follows.

Although LTE OLPC employs a combination of open-loop (OL) and closed-loop (CL) control to set the UE transmit power (up to a maximum level of $P_{MAX} = 24$ dBm) as follows:

$$P^{\text{UE}} = \min \left[P_{MAX}, \underbrace{P_0 - \alpha \cdot G}_{\text{OL operating point}} + \underbrace{\Delta_{\text{TF}} + f(\Delta_{\text{TPC}})}_{\text{dynamic offset}} + \underbrace{10 \cdot \log_{10} M}_{\text{BW factor}} \right],$$

where G is the path gain between the UE and the BS, for PPC we set the CL component (dynamic) to zero. ² The OL operating point allows for *path loss (PL) compensation* while the bandwidth factor takes into account the number of scheduled resource blocks(M). For the OL operating point, P_0 is a base power level used to control the SNR target and it is calculated as:

$$P_0 = \alpha \cdot (\gamma^{tgt} + P_{IN}) + (1 - \alpha) \cdot (P_{MAX} - 10 \cdot \log_{10} M),$$

where α is the PL compensation factor and P_{IN} is the estimated noise and interference power.

B. Greater Than 1 Pilot Reuse Schemes

Full pilot reuse (i.e. reuse-1 of pilot sequences) leads to high inter-cell interference during channel estimation, which can be mitigated using a less aggressive pilot reuse factor. Pilot reuse schemes specifically in the context of massive MIMO systems have been studied by, for example, [4] and [8].

Pilot reuse is analogous to traditional *frequency reuse* in the sense that terminals within the pilot reuse area are confined to utilize only a fraction of the time-frequency resources during the channel estimation phase. However, with pilot reuse, each terminal is free to use all the available resources for data transmission during the rest of the coherence interval. The pilot reuse factor 1/U is the rate at which pilot resources may be reused in the network, where U is the number of cells that are assigned orthogonal pilots. A factor U > 1 always reduces the pilot contamination effect by assigning orthogonal pilots to

neighboring cells, the next-neighbor cells and so on. The total number of unique time-frequency elements reserved for pilot transmission are $K \cdot U$, where K is the number of terminals per cell.

In this work we consider a hexagonal network layout, in which case the smallest reuse factor that ensures orthogonal pilots in adjacent cells is U = 3 To implement this reuse factor, 3K time-frequency resources are required to generate the U = 3 groups of K mutually orthogonal pilot sequences in each group. With pilot reuse, a pilot group is assigned to each cell according to the reuse pattern, and the pilots within that group are randomly distributed to the terminals as in the case of full pilot reuse.

C. Coordinated Pilot Allocation (CPA)

The CPA algorithm aims at identifying the set of users who are spatially well-separated and are therefore better candidates to reuse identical pilot sequences in the L-cell system without causing significant pilot contamination to one another [6]. Specifically, it defines the network utility function

$$F\left(\mathcal{U}\right) \triangleq \sum_{l=1}^{|\mathcal{U}|} \frac{\mathcal{M}_{l}(\mathcal{U})}{\operatorname{tr}\{\mathbf{R}_{ll}\left(\mathcal{U}\right)\}}$$

where we used the notations defined in Table I and $|\mathcal{U}|$ is the cardinal number of the set \mathcal{U} . $\mathcal{M}_l(\mathcal{U})$ is the estimation mean square error (MSE) for the desired channel at the *l*-th base station and $\mathbf{R}_{ll}(\mathcal{U})$ is the covariance matrix of the desired channel at the *l*-th cell.

TABLE I Notations Used by the Coordinated Pilot Allocation (CPA) Scheme

Notation	Meaning
$K_l(p)$	Index of the User in Cell- <i>l</i> who is assigned the p^{th} pilot sequence (i.e. \mathbf{s}_p)
$\mathcal{U}(p)$	Set of Users using the p^{th} pilot sequence (i.e. contaminating each other in the <i>L</i> -cell system)
$\mathcal{G}_l(p)$	Set of Users in Cell- l who are not (yet) assigned a pilot sequence after assigning the first p pilot sequences to users.

For each pilot sequence s_p , CPA attempts to identify the set of users who, when reusing this same s_p pilot sequence in each cell, minimize the sum MSE metric as indicated by Line 4 of the CPA Algorithm below. Initially, for p = 1, the set of users in each cell out which CPA can find the spatially most separated user (in sum MSE sense) comprises all users of that cell, that is $\mathcal{G}_l(1) \equiv \{1, \ldots, K\}$.

It is important to realize that as CPA progresses from $p = 1, \ldots, P$, the set of users not yet assigned a pilot sequence shrinks and thereby the possibility of finding spatially well separated users gradually vanishes. In fact, as we will see in the numerical section, the MSE performance of CPA severely degrades already after the first few users get assigned their pilot sequences.

²We leave the investigation of employing the closed loop component for pilot power control for future work.

Algorithm 1 Coordinated Pilot Allocation (CPA)

1:	for $p = 1,, P$ do
2:	$\mathcal{U}(p) := \emptyset$
3:	for $l = 1, \ldots, L$ do
4:	$\mathcal{K}_l(p) = \arg\min_{k \in \mathcal{G}_l(p)} F(\mathcal{U}(p) \bigcup \{k\})$
5:	$\mathcal{U}(p) \leftarrow \mathcal{U}(p) \bigcup \{\mathcal{K}_l\}$
6:	$\mathcal{G}_l(p) \leftarrow \mathcal{G}_l(p) \setminus \{k\}$
7:	end for
8:	end for

IV. NUMERICAL RESULTS

We consider a 7 site TDD system with inter-site distance of 500m, in which each site accommodates 3 cells (sectors). Each BS, located at the center of its site, is equipped with M = 20...100 antenna elements arranged in a uniform linear array with antenna spacing 0.7 λ . In this system we let a varying number (K = 3...24) of mobile users with vehicular speed of 60 kmph transmit UL pilot signals to facilitate CSI acquisition at the BS. The BS uses LS or MMSE channel estimation and MRT or ZF precoding to transmit in the DL. To gain insight into the performance implications of pilot reuse and pilot power control, we are interested in the NMSE of the estimated channel and the resulting SINR and DL sum rate when the pilot sequences are assigned randomly or according to the pilot coordination algorithm of [6]. The system parameters are summarized by Table II.

TABLE II Simulation Parameters

Scenario	ITU Urban Macro
Network Deployment	21-cell hexagonal grid
Inter-site distance	500 m
Exclusion radius	35 m
Terminals per cell (K)	{3,6, 12 ,24}
Terminal speed	60 kmph
BS transmit power (P_{BS})	0.067W per subcarrier
Max terminal transmit power $(P_{\rm T})$	23 dBm over 20 MHz
Carrier Frequency (f_c)	2 GHz
Subcarrier spacing	15 KHz
BS array	100-antenna uniform linear array
Tilt	11°
BS antenna	Fitted Kathrein, Vert. Polarized
Antenna spacing	$0.7 \lambda_c$
Max. antenna gain	18 dBi
3dB horizontal beamwidth	65°
3dB vertical beamwidth	6.5°
BS antenna noise figure	5 dB
Terminal antenna	Omnidirectional, Vert. Polarized
Terminal antenna noise figure	9 dB

Figure 1 compares the performance of pilot based channel estimation when employing maximum pilot transmit power and full pilot reuse (U = 1) with a system that uses OLPC of the pilot signals or employs a greater than 1 reuse factor (U = 3) for the pilots. The figure shows that the NMSE of users with very channel estimation errors can be much improved by employing OLPC for the pilots, although this performance improvement happens at the expense of increased channel estimation error of users with low NMSE. On the



Fig. 1. The CDF of the normalized mean square error (NMSE) of the estimated channel using LS and MMSE estimation. We can see that pilot reuse (with a reuse factor of U = 3) drastically reduces the estimation error of either LS or MMSE estimation with full pilot reuse.

other hand, pilot reuse can dramatically improve the estimation performance in the entire regime of the NMSE CDF.



Fig. 2. The average NMSE as a function of the number of BS antennas. The average NMSE can be largely reduced by means of pilot power control (OLPC) or pilot reuse (U = 3) independently of the number of BS antennas. The NMSE can also be improved by MMSE, but this improvement is much dependent on the number of BS antennas.

Figure 2 focuses on the *average* NMSE as a function of the number of BS antennas and shows the impact of OLPC and pilot reuse. This figure reinforces the insight of Figure 1 by showing the superior estimation performance of power controlled pilots and especially of higher pilot reuse for all number of antennas (M = 2...100).

Figure 3 compares the achieved DL SINR with OLPC and pilot reuse. Pilot reuse with U = 3 practically realizes the performance of a system with perfect CSI, whereas OLPC performs close to it. The high DL SINR values achieved by OLPC may be somewhat surprising when comparing the average NMSE results of Figure 2, but we recall from Figure 1 that OLPC drastically improves the poor estimation region (NMSE above 0 dB) that is decisive for the SINR performance.

Figures 4-8 show the DL sum rate and thereby give insight into the trade-off between the pilot reuse factor and the available symbols for data transmission. First, Figure 4 shows the sum rate loss due to imperfect CSI whith MRT



Fig. 3. Comparing the impact of OLPC and pilot reuse on the achieved DL SINR in the case of MRT precoding. Pilot reuse (U = 3) practically achieves the DL SINR performance of a pilot contamination free system, whereas the performance of a system employing OLPC is somewhat worse.



Fig. 4. The impact of channel estimation error on the DL sum rate with MRT (left) and ZF (right) precoding as a function of the number of antennas. We see that pilot contamination severely degrades the performance for all K (number of served users) values and practically eliminates the performance advantages of ZF over MRT precoding.

and ZF transmission when the number of served users varies between K = 3 and K = 24. Imperfect CSI severely degrades the performance (especially with ZF precoding) practically rendering the performance of MRT and ZF similar in terms of DL sum rate.

This insight is reinforces and extended by Figure 5 that shows that the rate performance of MRT and ZF transmission gets very similar with imperfect CSI, although MMSE channel estimation (with MRT precoding) results in best performance for all number of antennas M = 20...100. Notice that for MRT, MMSE estimation performs close to the performance of a system with perfect CSI.

The effect of CPA on channel estimation is shown in Figure 7. Clearly for the *initial* pilot allocations, there is large 'pool' of users to choose from when allocating pilots, and it is possible to allocate pilots in the neighboring cells to spatially well-separated users. However, for subsequent pilot allocations, the greedy algorithm is constrained to allocate pilots to users who might be spatially close, leading to a high estimation NMSE.

In Figure 8, we observe that for the overall system, the



Fig. 5. Comparing the impact of channel estimation error on the DL sum rate when using LS or MMSE estimation and MRT or ZF precoding. We can see that with imperfect CSI, the performance gap of these different techniques is much less than with perfect CSI.



Fig. 6. The impact of reducing the channel estimation error by means of the pilot reuse scheme with U = 3 on the achieved DL performance with MRT and ZF precoding (and LS estimation). We can see that improving the quality of CSI always improves the sum-rate and is especially beneficial with ZF.

coordinated approach is unable to provide any rate gains over random pilot allocation, for both MRT and ZF precoders.

V. CONCLUSIONS

In this paper we proposed a proposed pilot contamination mitigation technique based on pilot power control and a pilot contamination avoidance (within a group of cells) scheme based on pilot reuse. We demonstrated that in spite of being simple and based on existing LTE measurements, this scheme is effective in mitigating pilot contamination. We compared the performance of coordinated pilot allocation scheme against random allocations. The performance measures of interest included the mean square error of the channel estimates and the DL spectral efficiency. Somewhat surprisingly we found that although pilot coordination can significantly improve the performance of the first few users, it does not provide performance gains over systems that use lower complexity pilot



Fig. 7. The performance of coordinated pilot allocation (CPA) in terms of average NMSE. For the initial pilot allocation it provides gains over random allocation, but at the cost of poorer channel estimation for further pilot allocations



Fig. 8. Comparing the performance of random and coordinated pilot allocation with MRT (left) and ZF (right) precoding. With MRT, coordinated pilot allocation cannot improve the performance of MMSE estimation. With ZF, coordinated pilot allocation is counter-effective (inexpedient) in terms of the achieved DL sum rate.

contamination mitigation or avoidance schemes. Specifically, we found that a pilot reuse-3 scheme strikes a good balance between the number of resources in terms of coherent symbols used for channel estimation and DL data transmission. Also, the well proven LTE OL power control scheme can significantly improve the quality of the channel estimation even under pilot reuse-1 and can approximate the performance of the contamination free system. The investigation of these algorithms under different channel conditions affecting the coherence budget is left for future work.

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